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Choosing the Discount Rate for Defense Decisionmaking

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Presents the results of research on the choice of the discount rate for the evaluation of public projects. The need for discounting arises in the evaluation of both public and private projects because costs and benefits occurring in different years must be treated differently. While all serious economists believe that discounting is the correct way to reduce a stream of costs or benefits to a single number so that one stream can be compared with another, there is much disagreement over the appropriate rate to apply in actual decisions. In evaluating public investments, particularly military projects, states of the world in which tastes, production possibilities, or benefits differ from those of the current state or most likely future state, are too often ignored. It should be possible to improve our assessments of military projects if the effect of possible countermeasures are directly incorporated into the present value calculations. (PB)

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PREFACE

This report presents the results of Rand-sponsored research on the choice of the discount rate for the evaluation of public projects. The immediate impetus for this research came from the observation that in several recent Rand publications the discount rate played an important part in the ultimate results. (Most recent among these publications is an economic analysis of shipboard automation, R-1790-ARPA,[1] in which the author, who is also the author of this report, chose to recommend a real discount rate of 10 percent.)

Rand's interest in this issue is longstanding[2] and derives from the fact that many decisions within DoD, involving billions of dollars of public funds, could be improved if the meaning of the discount rate and of the uses of discounting were better and more widely understood. The objective of this report is to provide a reasonably comprehensive discussion of the issues that surround the choice of the discount rate, to compile a bibliography of material on the discount rate, and to investigate the complications that occur when an opponent actively seeks to reduce the benefits of a proposed project.

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[1] Robert Shishko, *The Economics of Naval Ship Automation: An Analysis of Proposed Automation of the DE-1052*, The Rand Corporation, R-1790-ARPA, November 1975.

[2] See, for example, Charles J. Hitch and Roland N. McKean, *The Economics of Defense in the Nuclear Age*, The Rand Corporation, R-346-PR, March 1960, especially pp. 205-218; also Gene H. Fisher, *Cost Considerations in Systems Analysis*, The Rand Corporation, R-490-ASD, December 1970, pp. 51-59 and 217-231.

SUMMARY

The need for discounting arises in the evaluation of both public and private projects because costs and benefits occurring in different years must be treated differently. While all serious economists believe that discounting is the correct way to reduce a stream of costs or benefits to a single number so that one stream can be compared with another, there is much disagreement over the appropriate rate to apply in actual decisions. In the public investment area, many economists harbor the suspicion that numerous government projects that would be rejected by the private sector are funded because the wrong discount rate is used.

The level of our defense effort--that is, what capabilities we wish to have over some appropriate time horizon--should not be governed by the discount rate. The discount rate only helps to choose the most efficient way of obtaining the capabilities we deem necessary.

A *real* rate between 8 and 10 percent can be supported from various studies. The studies by Harberger (1968) and Stockfisch (1969), in which these economists estimated the social opportunity cost of capital, are probably the best. In Harberger, the social opportunity cost of capital is a weighted average of the after-personal-income-tax rate of return to savers and the pre-corporate-income-tax cost of capital. Stockfisch calculates the pre-tax rate of return in several corporate sectors and takes a weighted average of that with the rate of return in the noncorporate sector.

The 10 percent figure suggested by many outstanding economists and enshrined in DoD and OMB directives is an informal estimate of the social opportunity cost of capital. In spite of the back-of-the-envelope nature of the estimate, *careful* studies like the two mentioned above have found 10 percent to be quite reasonable.

Defense investments, like private investments, have different payoffs depending on the state of the world that actually materializes. The unique feature of defense investments is that the return is partially under the control of an opponent who is actively seeking to reduce the payoff through countermeasures. (This is in contrast to private

investments, which face only a benign Nature.) Techniques like the state-preference approach can be used to handle this kind of uncertainty.

The choice of the discount rate is just a part of the larger issue. In evaluating public investments, particularly military projects, states of the world in which tastes, production possibilities, or benefits differ from those of the current state or most likely future state are too often ignored. It should be possible to improve our assessments of military projects if the effect of possible countermeasures is directly incorporated into the present value calculations.

ACKNOWLEDGMENTS

The author wishes to thank Rand colleagues Kenneth J. Arrow, John J. McCall, Jack Hirshleifer and Richard V. L. Cooper for their comments during the preparation of the report. The views expressed are, of course, solely the author's.

CONTENTS

I. DISCOUNTING IN DEFENSE DECISIONMAKING

Discounting in the defense decisionmaking environment is the principal concern of this report. The need for discounting arises in the evaluation of both public and private projects because costs and benefits occurring in different years must be treated differently. Discounting at the correct rate enables the analyst to transform dollar amounts from different years into dollar amounts of a common year, thus facilitating aggregation. If the common year chosen is the current year, this aggregation is called the present discounted value or just present value. The discount rate discussed in this report refers to the rate to be applied to monetized values--that is, to cases in which the costs and benefits can be measured in homogeneous units.[1]

One source of confusion on discounting in the defense environment is the apparent belief by some that the selection of the discount rate is tantamount to choosing the level of defense activity. This is not the case, because of the difficulty in translating the output of a particular military system into a dollar value with which the cost of the system can be compared. *If* such a metric existed, the optimal level of defense activity could be determined objectively as a function of the technological opportunities, factor prices, and the discount rate, much as the optimal output of some good is determined by market forces.

That defense activities are not marketed in the economic sense implies that there is no "natural" means of determining "how much is enough." The level of defense activity is, and should be, the result of the continuous political process out of which the collective wisdom of the legislative and executive branches and their respective constituencies among the electorate emerges. My point is simply that once a

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[1]Further, the discount rate discussed in this paper is the rate for public decisionmaking. Occasionally this discount rate is used interchangeably with the "individual discount rate" and the "interest rate." Although individual discount rates are useful in predicting individual behavior, and one can speak of the interest rate in a particular market as the result of the market behavior of many individuals, these notions are conceptually different from the discount rate for public projects.

separate decision has been made as to the level of net defense capability--that is, effectiveness over some time horizon--*then* computing present values using the right discount rate should reveal the *most efficient way* of achieving that effectiveness.

Let me give an example of when present value analysis should be used. Suppose a decision has been made to increase the effectiveness of our tank forces. We could invest in reliability increases for tanks or we could add enough tanks of the unimproved kind to just give the same increase in force effectiveness. By discounting the streams of capital investment, labor, and other costs at the correct rate, we could discover which of the alternatives had the higher present discounted value. Both should have negative present discounted values because we are genuinely increasing our capability, but our only concern is choosing the least expensive way of getting that capability.

Alternatively, suppose that the original level of effectiveness was desired; we could reduce the number of tanks by improving the reliability of the remaining force, or we could stick with the original force. Discounting should be used to determine whether the reliability improvement makes sense, that is, whether it has a positive present discounted value.

What the present discounted value of the reliability improvement alone does not tell the DoD manager is whether to take any cost savings in the form of additional effectiveness or to turn those savings back to society. That is quite a different kind of decision. Calculating the present discounted value tells the DoD manager only whether there are savings to be had.

A second source of confusion to some concerns the difference between the nominal and real discount rates. In calculating the present discounted value, there are two ways to go; both will yield the same results. First, one can deal in real dollars (dollars adjusted to some fixed price level) and discount by the real rate, or one can deal in nominal dollars (sometimes called then-year dollars) and discount the nominal discount rate. Let ρ be the real discount rate, r the nominal discount rate, and θ the constant (expected) rate of inflation, then $r \simeq \rho + \theta$. The effect of discounting nominal dollars by the nominal

discount rate is first to deflate all then-year dollars to present-year (constant) dollars and then to discount by the real rate. If the base year is the same, both ways of calculating present discounted value yield identical results.[1]

In the next sections, the question of what real discount rate to use in the evaluation of military investments is addressed. The reader should keep in mind that these investments are of the kind that "re-arrange" cost streams for equally effective systems. The "benefits" of these investments are then the savings that might accrue over time relative to some "base case" system.

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[1] *Relative* price changes are important. For example, the cost of military manpower might be expected to rise faster than the overall price index. This is fairly easy to take into account in making present discounted calculations. For example, see Robert Shishko, *The Economics of Naval Ship Automation: An Analysis of Proposed Automation of the DE-1052*, The Rand Corporation, R-1790-ARPA, November 1975.

II. ISSUES IN THE CHOICE OF THE DISCOUNT RATE

Discounting--the technique by which resources produced or consumed in different time periods can be made commensurable--has been the subject of much debate within the economics profession. While all serious economists believe that discounting is the correct way to reduce a stream of costs or benefits to a single number so that one stream can be compared with another, there is much disagreement over the appropriate rate to apply in actual decisions.[1] In the public investment area, many economists harbor the suspicion that numerous government projects that would be rejected by the private sector are funded because the wrong discount rate is used.

A project that shows a positive present discount value (PDV) at a 5 percent discount rate may show a negative PDV at a 10 percent discount rate.[2] At stake, then, in the choice of the discount rate may very well be the acceptance or rejection of a particular project even when all are agreed on the costs and benefits of the undertaking. On the macro level, at stake is the division between public and private capital formation--not an insignificant matter.

For the moment let me ignore the problems of risk. There are basically two views on how the discount rate ought to be selected. The first view is that the discount rate ought to reflect the (social) opportunity cost of capital, which is also known as the intertemporal marginal rate of transformation (MRT). According to this view, which is held by economists like Baumol, Hirshleifer, Harberger, and Stockfisch, only by discounting future costs and benefits at the rate that could be earned by the best alternative private project can society

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[1]Throughout this report, all costs and benefits have been monetized so as to avoid the problems of having benefits and costs measured in different units.

[2]It is possible that when comparing two projects A and B, project A is preferred at one discount rate and project B is preferred at another rate. This anomaly can arise when the time streams of net benefits are not even or not monotonically rising or falling the same way.

be guaranteed that a public undertaking does not displace a private undertaking that yields more. Central to this viewpoint is that the (social) opportunity cost of capital can in fact be measured. There have been a number of attempts to measure the opportunity cost of capital; the most widely acknowledged of these are separate studies by Harberger,[1] Stockfisch,[2] and Haveman.[3] The basic methodology is to make an assumption about how the marginal dollar of public funds will be raised--that is, by borrowing or taxing--and to estimate the incidence of the additional taxes or borrowing on various capital-using sectors. The estimates of incidence provide the weights by which the pre-tax rate of return on capital in each sector is multiplied.[4] A number of difficulties in implementation can be identified. The first, which has already been mentioned, is deciding where the additional public resources will be drawn from--that is, what portion of public resources will be taken from business investment and what portion from private consumption. The method of financing the additional public revenue will of course alter the portion taken from businesses and consumers.

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[1]Arnold Harberger, "On Measuring the Social Opportunity Cost of Public Funds," *The Discount Rate in Public Investment Evaluation*, Report 17, Conference Proceedings from the Committee on the Economics of Water Resources Development of the Western Agricultural Economics Research Council, Denver, Colorado, December 17-18, 1968.

[2]Jacob Stockfisch, "Measuring the Opportunity Cost of Government Investment," Research Paper P-490, Institute for Defense Analyses, March 1969.

[3]Robert H. Haveman, "The Opportunity Cost of Displaced Private Spending and the Social Discount Rate," *Water Resources Research*, Vol. 5, October 1969, pp. 947-957.

[4]The existence of different sectoral rates of return implies either differences in risk among sectors, difference in taxes, or imperfections in the capital market. In the three studies cited above, differences in risk have not been purged from the sectoral rate-of-return data. Consequently, the estimates of social opportunity cost of capital include risk premia.

The primary difference in taxes occurs between the corporate and noncorporate sector. Many economists assume 100 percent shifting of the corporate income tax, so if the corporate income tax is 50 percent, then the rate of return in the corporate sector is twice that of the noncorporate sector.

What years to include in one's data is a second difficulty as is the calculation of each sector's rate of return. This calculation depends heavily on what assumptions are made about the stock of capital in each sector and on the effect of various income, excise, and property taxes. In particular, the calculation of depreciated capital assets presents a problem because accounting depreciation for tax purposes, sinking fund contributions for replacement investment, and economic depreciation (the loss of market value of physical capital) need not be the same.

These difficulties aside, separate studies by Harberger, Stockfisch, and Haveman give different results because different assumptions are made by each author. In Harberger, the social opportunity cost of capital is a weighted average of the after-personal-income-tax rate of return to savers and the pre-corporate-income-tax cost of capital. Stockfisch calculates the pre-tax rate of return in several corporate sectors and takes a weighted average of that with the rate of return in the noncorporate sector. Haveman assumes that additional government revenue will be financed completely through the personal income tax, on which the relevant rate of return is a weighted average of various consumer borrowing rates.

The second major view on the discount rate, associated with economists such as Marglin, Feldstein, Somers, and Bradford, is that one should use society's rate of time preference, which is also called the intertemporal marginal rate of substitution (MRS).[1] In a Fisherian world with no taxes, externalities, or market imperfections to drive a wedge between society's MRT and MRS, the opportunity cost rate and time preference rate would be the same. In the real world, taxes, differential costs of information, and monopolies act to create a difference between society's opportunity cost rate and rate of time preference. Some adherents to the time preference view suggest that society's MRS can be

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[1]Heuristically speaking, the intertemporal MRT is the most efficient rate at which society is able to transform resources today into resources tomorrow, whereas the intertemporal MRS is the rate at which society is willing to forgo resources today for resources tomorrow leaving utility unchanged.

inferred from households' decisions regarding savings and consumption or borrowing and lending. Other economists suggest that the MRS is different for different classes of projects and that for a particular class it is whatever society wants it to be. A less "flexible" group of economists suggests that it can be inferred for a class of projects from past voter-consumer referenda by whether such projects were accepted or rejected.

A strong case for ignoring current market decisions by individuals has been made in separate articles by Marglin[1] and Feldstein.[2] In essence these authors believe that individuals are irrationally myopic, that future generations are under-represented in current capital markets, and that society, acting collectively, may (should) desire a distribution of income among generations different from that it generates through individual behavior. Accordingly, it would not be inconsistent to borrow at say 15 percent to increase current consumption while voting increased taxes for a project yielding 8 percent because by calling upon the government's power to tax, the individual can essentially guarantee that the other individuals in society will be compelled to contribute to the project as well.

If the argument is accepted, then the proper policy is to lower the market rate(s) of interest for all investments using monetary and fiscal instruments. At the lower rate of interest, the rate of return required by investors would also be lower, leading presumably to the acceptance of some projects that were previously rejected. If monetary and fiscal policies can be used to reduce the interest rate (or interest rates), then a separate social discount rate is unnecessary; but if the use of monetary and fiscal policies is inhibited, then a "second-best" policy may be to use a social discount rate lower perhaps than either the MRT

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[1]Stephen A. Marglin, "The Social Rate of Discount and the Optimal Rate of Investment," *Quarterly Journal of Economics*, Vol. 77, February 1963, pp. 95-112.

[2]Martin S. Feldstein, "The Social Time Preference Discount Rate in Cost-Benefit Analysis," *Economic Journal*, Vol. 74, June 1974, pp. 360-379.

or MRS in the evaluation of public projects. Such a move, as Hirshleifer points out, would be extreme.[1]

Despite the appearances to the contrary, the two views are not devoid of overlap. Harberger's calculation of the social opportunity cost of capital includes the after-personal-income-tax rate of return on savings, which is presumably related to the rate of time preference.[2] Using a two-period model, several authors[3] have shown that with no externalities in either government or private investment in the first period, the appropriate discount rate is a weighted average of the intertemporal MRT and intertemporal MRS, the weight attached to the MRT being the proportion of the marginal dollar of government investment that is drawn away from private investment. On this last point, some economists have argued that government investment in fact produces positive spillovers on private investment. They argue essentially that although a dollar of government investment may displace some private capital formation, the effect of the flow of services from the government project may be to increase downstream private capital formation. In other words, the output flow of government investments shifts the entire investment curve to the right. This concept is particularly appealing if government investment is different in character from private investment.[4] As an example, one might expect some positive effects

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[1]See Jack Hirshleifer, "Social Time Preference," Discussion Paper 18, Department of Economics, University of California at Los Angeles, April 1972.

[2]Harberger's calculated social opportunity cost of capital is in effect a weighted average of the rate of return on capital and the after-tax interest rate on savings. Because individuals save for reasons not confined to the desire to optimize consumption streams, I am in doubt as to whether the after-tax interest rate on savings is the social rate of time preference. It seems more likely that the after-tax interest rate on consumer borrowing is closer to the social MRS. Stockfisch, on the other hand, calculates a weighted average of the rate of return on capital in the corporate and noncorporate sectors. His implicit weight of zero on an MRS component explains why his estimate is higher than Harberger's.

[3]See separate articles by Bradford, Diamond, and Sandmo and Drèze.

[4]Indeed, legal restrictions on the kinds of investments the government may undertake tend to reinforce the dissimilarities between government and private investments.

on private investment from government investment in the economic infrastructure. Under the assumption that the output from an additional dollar of government investment increases private investment by exactly the same amount that private investment is decreased as a result of government financing, then the appropriate discount rate according to the hybrid view is the intertemporal MRS. However, under the assumption that an additional dollar of government investment displaces one dollar of private investment with no positive external effect in the other direction, then the government should use a discount rate equal to the intertemporal MRT. This is perhaps best represented by the case in which a government project is a perfect substitute for a private project.

Having dealt with the theory of the discount rate a bit, let me turn to actual recommendations. Some of the nominal discount rates in Table 1 were calculated from data; the origin of the DoD and OMB recommended rates is less clear. In all applicable cases, I calculated a real discount rate by subtracting a geometrically computed average of the inflation rate during the six years prior to the year of the estimate.

The range of the recommended real discount rates, which is perhaps still too large for some, results of course from the different assumptions made by each author. Haveman's 6 percent recommendation might be favored by those who adhere to time preference theory. Estimates between 7.5 percent and 10 percent probably reflect weightings of the opportunity cost and time preference rates. A real rate between 8 and 10 percent seems to be justified on the basis of Table 1.

I have left the problems of risk and the discount rate until last primarily because the next two sections deal with the theory and alternative practices in greater detail. The major issue is whether private risk is a social risk as well. The argument that a private risk is not a social risk is that when the risks associated with individual projects are pooled and averaged over the entire population, the social risk approaches zero in the limit. Therefore, for purposes of calculating the social opportunity cost, one should use the rate of return less the risk premium.

The counterargument runs as follows: The pooling of risks is not sufficient to reduce the social risk to zero. A necessary condition

Table 1

RECOMMENDATIONS ON THE DISCOUNT RATE

Author	Year[a]	Recommended Nominal Rate (%)	Adjusted for Expected Inflation[b] (%)
Krutilla and Eckstein	1958	6.0	4.58
Hirshleifer, DeHaven, and Milliman	1960	10.0	8.39
Bain, Caves, and Margolis	1966	6.0	4.65
Haveman	1966	7.3	5.95
DoD Directive[c]	1966	--	10.00
Stockfisch[d]	1949-1965	12.0	10.67
Harberger	1968	10.68	8.33
Baumol	1968	10.0	7.65
OMB Directive[e]	1972	--	10.00
Dorfman	1975	(f)	7.50

[a]This column refers to the year (or years) to which the recommended nominal rate applies and not necessarily to the year of publication of the recommended nominal rate.

[b]The adjustment for (expected) inflation was made by calculating a geometric average of the rates of inflation in the six years prior to the year of the estimate and subtracting it from the nominal rate. This geometric average rate of inflation θ^t was calculated from the equation

$$1 + \theta^t = \prod_{j=t}^{j=t-5} (1 + \theta_j)^{1/6},$$

where θ_j is the rate of inflation in year j and t is the year of the estimate.

[c]DoD Instruction 7041.3, December 19, 1966.

[d]If the anticipated rate of inflation were calculated using the entire 1949-1965 period, the adjusted recommended rate would be 10.4 percent.

[e]OMB Circular No. A-94, March 27, 1972.

[f]Dorfman's estimate is based almost completely on theoretical considerations. The estimate shown in the table, however, relies on parameters derived from U.S. experience in the 1960s.

for a zero social risk is a zero average covariance among the rates of return; this, of course, can occur if the rate of return on each project is an independent random variable, or if there is a significant negative covariance among some projects. The existence of business cycles is some indication that individual rates of return are in fact positively correlated. In other words, if the rate of return to a particular project is correlated with national income--not an unreasonable assumption for most projects--the social risk cannot be zero; after all, monetary and fiscal policy are not perfect instruments of national income management.

Furthermore, economists opposing the use of a riskless discount rate correctly observe that the private investor can diversify his portfolio at negligible marginal cost by participating in markets for fractional claims on a wide variety of private projects--that is, stock markets. The individual can reduce his private risk to the average covariance among projects, which is an irreducible social risk. Therefore, the pooling argument is valid only if the government can provide more efficient diversification than can private markets for risk-bearing.[1]

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[1]Bailey and Jensen argue that the government is even less able to distribute risks than are private markets. In the case where both private risk markets and the government are imperfect distributors of risk--the most likely case according to Bailey and Jensen--then the risk premium for public projects must be the same as that demanded in the private sector for bearing that risk. See Martin J. Bailey and Michael C. Jensen, "Risk and the Discount Rate for Public Investment," in *Studies in the Theory of Capital Markets*, Michael C. Jensen (ed.), Praeger, New York, 1972.

III. THE STATE-PREFERENCE APPROACH TO THE CHOICE OF THE DISCOUNT RATE

The state-preference approach is ideally suited for an analysis of the appropriate discount rate under uncertainty. Under the state-preference approach, the outcome of a given investment--for example, the benefits of a particular public project--in any subsequent period depends on the state of the world in that period. The state of the world in some future period is of course uncertain, but under the state-preference approach, it is assumed that all possible future states can be enumerated. Further, those future states are assumed to be mutually exclusive. These two assumptions are not altogether unreasonable if the relevant states of the world are vastly different from one another, as for example, war versus peace, or prosperity versus depression.

A private investor considering a potential investment will rationally want to contemplate the return he will obtain in each of the relevant future states. The income resulting from this investment will generally be different in each of these future states, and therefore we may picture the risk-averse investor as willing to exchange his claims on future income in some states for claims on future income in other states. The establishment of markets for various contingent claims on future income will enable the private investor to make such trades and achieve his desired diversification. A perfectly competitive market in contingent claims on future income produces the usual Pareto efficiency.[1] The independent trading decisions by many individuals establish a set of prices (to be paid now) for one dollar of income in each of many future states of the world. These prices naturally reflect the market's collective wisdom about the probabilities of each of the relevant states, the relative desirability of income in each of the states, and time-preference.[2]

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[1]In the private sector, the operation of various securities, commodities, and insurance markets partially serves this laudable purpose.

[2]In his classic 1966 article, Hirshleifer shows how an individual, facing a set of prices for contingent claims that he is powerless to affect, should rationally choose an investment portfolio. See Jack

Using the prices established for contingent claims, it is possible to specify a decision rule for government investments that generalizes the familiar present value criterion from the riskless to the risky case. This rule is that the government ought to do all projects whose present certainty-equivalent value (PCEV) is greater than zero. The PCEV is given by

$$V_0 = -p_0 C_0 + \sum_{j=1}^{T} \sum_{i=1}^{n_j} p_{ij} S_{ij} \tag{1}$$

where V_0 is the PCEV, p_0 is the price of current claims generally taken to be one, and C_0 is the current cost of the project. S_{ij} is the net dollar benefits occurring in the state i and time period j, and p_{ij} is the state-time price to be paid now to obtain one dollar in state i and time period j.

A simple example may help illustrate this rule. Suppose there is one future period in which one of two possible states must occur, e.g., war or peace. Suppose further that all costs are incurred in the present and all benefits are realized in the future time period. The PCEV is then given by

$$V_0 = -C_0 + p_{1a} S_{1a} + p_{1b} S_{1b} \tag{2}$$

where C_0 is the cost of the project, S_{1a} and S_{1b} are the benefits occurring in state a and state b, p_{1a} and p_{1b} are the prices for future contingent claims. The price of current claims is taken as the numeraire.

It is possible to express the price of a *certain* future claim as p_1 by observing that such a claim can be purchased for $p_{1a} + p_{1b}$. Alternatively, if $S_{1a} = S_{1b}$, there is no uncertainty and again the contingent price is $p_{1a} + p_{1b}$. This leads directly to the definition of a riskless discount factor, ρ_1, in a world with uncertainty,[1] namely

Hirshleifer, "Investment Decisions Under Uncertainty: Applications of the State Preference Approach," *Quarterly Journal of Economics*, Vol. 80, May 1966, pp. 252-277.

[1]The discount factor ρ_1 is just one plus the discount rate.

$$\frac{1}{\rho_1} = p_1 = p_{1a} + p_{1b} \; . \tag{3}$$

A simple model described by Professor Dan Usher in his 1969 communication to the *American Economic Review* can be adapted to bring the state-preference approach to the choice of the discount rate into sharper focus.[1] Suppose again there is one future period and two possible states a and b for this period. Let

$$T^a = T^a(C_{0p}, C_{0g}, C^a_{1p}, C^a_{1g}) = 0 \tag{4}$$

be the production possibilities curve if the future state a occurs, and let $T^b = 0$ be the same for future state b. C_{0p} is the current consumption of private goods; C_{0g} is the current consumption of public (government) goods. C^a_{1p} is the future consumption of private goods in state a; and C^a_{1g} is the future consumption of public goods in state a.[2]

Let $U^a(C_{0p}, C_{0g}, C^a_{1p}, C^a_{1g})$ be society's utility function in state a and U^b be the same in state b. The government is assumed to maximize the utility function in each state subject to the transformation function. Thus if state a occurs, the government would want to solve

$$\max \;\; U^a(C_{0p}, C_{0g}, C^a_{1p}, C^a_{1g})$$

subject to (5)

$$T^a \geq 0$$

$$C_{0p}, C_{0g}, C^a_{1p}, C^a_{1g} \geq 0 \; .$$

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[1]See Dan Usher, "On the Social Rate of Discount: Comment," *American Economic Review*, Vol. 59, December 1969, pp. 925-959.

[2]By making the marginal rate of transformation between private and public goods unity in both periods, the transformation function can be written as $T^a(C_{0p} + C_{0g}, C^a_{1p} + C^a_{1g}) = 0$ and similarly for T^b.

The first-order conditions for this maximization are:

$$\frac{U^a_{0p}}{T^a_{0p}} = \frac{U^a_{0g}}{T^a_{0g}} = \frac{U^a_{1p}}{T^a_{1p}} = \frac{U^a_{1g}}{T^a_{1g}} \tag{6}$$

where the subscripts on U^a and T^a indicate partial differentiation with respect to the appropriate consumption variable.

The marginal rate of substitution between public goods now and public goods in future state a is given by

$$\rho^a_{gu} = \left.\frac{dC^a_{1g}}{dC_{0g}}\right|_U = \frac{\dfrac{\partial U^a}{\partial C_{0g}}}{\dfrac{\partial U^a}{\partial C^a_{1g}}} . \tag{7}$$

This number, ρ^a_{gu}, gives the rate at which society would be willing to trade government goods now for government goods next period if state a occurs. Society's welfare is maximized in state a when

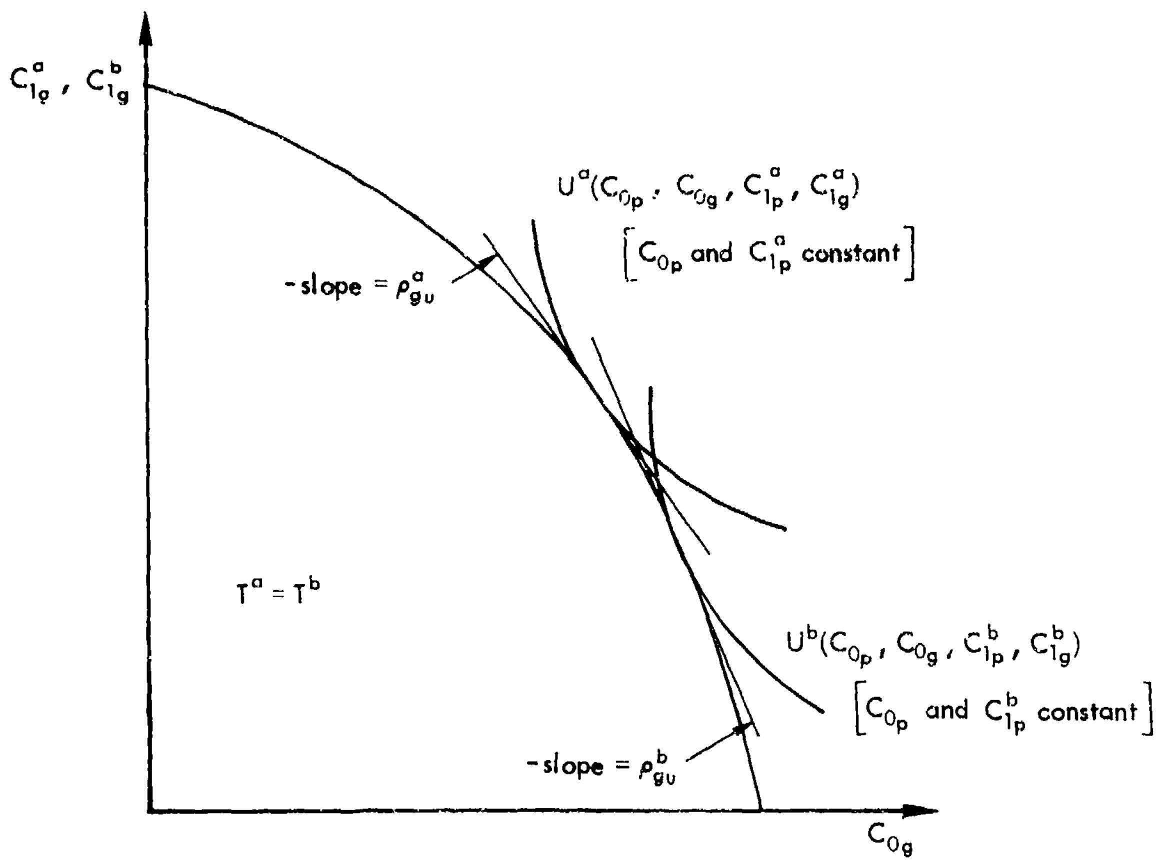
$$\rho^a_{gu} = \frac{\dfrac{\partial T^a}{\partial C_{0g}}}{\dfrac{\partial T^a}{\partial C^a_{1g}}} = \left.\frac{dC^a_{1g}}{dC_{0g}}\right|_T \tag{8}$$

as seen by rearranging Eq. (6).

By assuming that C^a_{1p} and C^a_{1g} are actually flows in perpetuity, ρ^a_{gu} takes on the character of an interest rate. In fact, I would define ρ^a_{gu} as the appropriate discount factor if it were certain that state a would occur. By Eq. (8), there is no divergence between society's intertemporal marginal rate of substitution and marginal rate of transformation of public goods now for public goods later, so the discount factor can be unambiguously defined.

By a similar process, the appropriate (riskless) discount factor for state b is ρ^b_{gu}. The key point is that ρ^a_{gu} is in general not equal

to ρ^b_{gu}. This is the essence of the state-preference approach. The marginal utilities of goods differ between states. The point can be demonstrated graphically in the figure below. Here I have made the important assumption that $T^a = T^b$--that is, that the production possibilities frontier is independent of the future state. I have also assumed that the government needs only choose the optimal amounts of C_{0g} and C^a_{1g} or C^b_{1g}; the private sector, it is assumed, can be relied upon to respond correctly to the resulting relative prices by supplying the optimal amounts of private goods in each period. The different utility functions U^a and U^b yield different intertemporal marginal rates of substitution and hence different discount factors. In the figure the ratio of the marginal utility of government goods now to the marginal utility



The social discount rate in states a and b when $T^a = T^b$

of government goods in the future in state b is greater than that ratio in state a. Consequently, society maximizes its welfare by consuming a larger amount of government goods now relative to state a *if state b were certain to occur.*

If $T^a \neq T^b$, it is possible for ρ^a_{gu} to equal ρ^b_{gu} if by pure coincidence the different utility functions and different production possibilities frontiers produce their respective tangencies at the same intertemporal marginal rate of substitution.

The relationship between this analysis and the state-preference approach boils down to the calculation of the prices for the contingent claims in states a and b. The usual tangency of the production possibilities frontier and highest attainable indifference curve yields the appropriate discount factor for each state if that state were certain to occur. Of course this is not the case--the future is not known with certainty. Each discount factor must be adjusted by the probability that that state will occur. Therefore, let π^a be the probability that state a will occur and $\pi^b = 1 - \pi^a$ be the probability that state b will occur[1]; then

$$p_{1a} = \frac{\pi^a}{\rho^a_{gu}} \tag{9a}$$

and

$$p_{1b} = \frac{1 - \pi^a}{\rho^b_{gu}} . \tag{9b}$$

[1] I have assumed that the probability of state a is objectively known and that society behaves like a von Neumann-Morgenstern maximizer.

IV. A COMPARISON OF VARIOUS DISCOUNTING PROCEDURES

We can now use the state-preference theory to compare various discounting procedures. To do this let me draw upon a recent study at Rand concerning the automation of the Navy's DE-1052 class destroyer escorts.[1] The issue is sufficiently simple: An investment in certain equipment for the DE-1052 will produce dollar savings by allowing for a reduction in shipboard manning. These savings are roughly proportional to the number of ships of the DE-1052 class that are so automated. Let us consider two possible future states of the world, peace and war. peace, which is also the current state of the world, continues through the next period, a certain level of savings will occur. If war occurs, the level of savings realized will undoubtedly be smaller because some portion of the ships will be lost in combat. However, a dollar's worth of savings may be valued differently in war than in peace. In particular, I will argue that in war a dollar's worth of savings will be valued higher because resources are scarcer. Even though the total dollar savings are less, the value attached to each dollar is higher so in essence these are partially offsetting effects.

If the probability of war is small, some analysts are inclined to ignore that possible state of the world and proceed to discount only the savings that occur in peacetime. Other analysts may calculate the *expected savings* and then discount. In fact I have identified five separate discounting procedures that collectively exhibit varying degrees of sophistication.

Let state a be peace and state war, and suppose the probability of war $1 - \pi^a$ is small. We may calculate the "present value" of the proposed automation of the DE-1052 by one of five following equations:

$$V_0 = -C_0 + \frac{S_{1a}}{\rho_1} \qquad (10a)$$

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[1] See Shishko.

$$V_0 = -C_0 + \frac{S_{1a}}{\rho^a} \quad (10b)$$

$$V_0 = -C_0 + \frac{\pi^a S_{1a} + (1 - \pi^a) S_{1b}}{\rho_1} \quad (10c)$$

$$V_0 = -C_0 + \frac{\pi^a S_{1a} + (1 - \pi^a) S_{1b}}{\rho^a} \quad (10d)$$

$$V_0 = -C_0 + \left(\frac{\pi^a}{\rho^a}\right) S_{1a} + \left(\frac{1 - \pi^a}{\rho^b}\right) S_{1b} \quad (10e)$$

where C_0 is the investment cost of the proposed automation; S_{1a} is the realized savings if future state a occurs; S_{1b} is the realized savings if future state b occurs; ρ_1 has been previously defined in Eq. (3) as the riskless discount factor; ρ^a and ρ^b have been previously defined except that for convenience I have dropped the subscript gu.

In Eqs. (10a) and (10b), the *most likely* benefits are discounted respectively by the riskless rate, and the (riskless) rate applicable to the *most likely* state. Equation (10c) is actually the procedure recommended by Arrow.[1] The expected savings are discounted by the riskless rate. In Eq. (10d), which is similar to Eq. (10c), the expected savings are discounted by the (riskless) rate applicable to the *most likely* state. Equation (10e) is the procedure recommended by Hirshleifer and is actually the PCEV. Here the savings in each state are valued by the prices for contingent claims in that state.

Equations (10c) and (10e) give the same answer if and only if the probabilities assigned to states are proportional to the prices of the contingent claims. From Eqs. (3) and (9) recall that

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[1]See K. J. Arrow and Robert C. Lind, "Uncertainty and the Evaluation of Public Investment Decisions," *American Economic Review*, Vol. 60, June 1970, pp. 364-378.

$$\frac{1}{\rho_1} = p_{1a} + p_{1b} = \frac{\pi^a}{\rho^a} + \frac{1 - \pi^a}{\rho^b} . \qquad (3')$$

If $\rho^a = \rho^b$ by some coincidence, then the above condition holds and the equality of Eq. (10c) and Eq. (10e) can be seen directly.

A numerical example will help illustrate how the choice of the discounting procedure can affect the decision to accept or reject the proposed project. Suppose the following values hold: $\pi^a = 0.9$, $C_0 = 1.1$, $S_{1a} = 1.50$, $S_{1b} = 0.50$, $\rho^a = 1.30$ and $\rho^b = 1.05$. In state b, war, I have assumed that only one-third of the DE-1052s will survive, so S_{1b} is only one-third of S_{1a}. From the information above, $\rho_1 = 1.27$. Table 2 presents the "present values" calculated from Eqs. (10a) through (10e).

Table 2

PRESENT VALUE OF HYPOTHETICAL SHIPBOARD AUTOMATION PROJECT USING FIVE DIFFERENT DISCOUNT PROCEDURES

Equation	V_0
10a	+ .081
10b	+ .054
10c	+ .003
10d	- .023
10e	- .014

If Eqs. (10a) and (10b) are used to evaluate the proposed automation, the project is accepted. If Eqs. (10d) and (10e) are used, the project is rejected; the project roughly breaks even if Eq. (10c) is used to evaluate costs and benefits.

In the example, state b, war, is the more highly valued but less likely state. The benefits in state b are lower than in state a. It is easy to show that if $\rho^a > \rho^b$ and $S_{1a} > S_{1b} > 0$, it is always true that V_0 evaluated by Eq. (10a) is greater than V_0 evaluated by Eq. (10b),

and that V_0 evaluated by Eq. (10c) is greater than V_0 evaluated by Eq. (10e), which is greater than V_0 evaluated by Eq. (10d).[1]

If $\rho^a > \rho^b$--that is, peace is the more highly valued state--then the relationships just described get turned around a bit. V_0 evaluated by Eq. (10b) is always greater than V_0 evaluated by Eq. (10a). If $S_{1a} > S_{1b} > 0$ again, then V_0 evaluated by Eq. (10d) is greater than V_0 evaluated by Eq. (10e), which in turn is greater than V_0 evaluated by Eq. (10c).[2]

The main point of my discussion is that the selection of the discount procedure is probably more important for the acceptance or rejection of a given project than the choice of the discount rate per se. What gets counted as part of the benefits in future periods and how those benefits are incorporated into the present value calculation are nonnegligible considerations. I need not have picked for my working example two such radically different states of the world as war and peace to illustrate this point for military investments. Consider for a moment, several "peaceful" states of the world, $a_1, a_2, \ldots, a_n$, in which the opponent has achieved various degrees of success in countermeasures. For example, in the case of the automation of the DE-1052, the opponent may have developed a way to make our DE-1052s so vulnerable that we voluntarily decide to retire them. The downstream benefits of automation will then not be realized. If the probabilities of achieving various degrees of success in countermeasures were known, we could treat this problem explicitly using the state-preference approach. However, the probability of the opponent achieving a countermeasure is *not*

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[1] If $\rho^a > \rho^b$, then it can be shown that $\rho^a > \rho_1 > \rho^b$, from which it follows that V_0 [10a] > V_0 [10b], V_0 [10c] > V_0 [10d], and V_0 [10e] > V_0 [10d]. (I have indicated the discounting procedure by which V_0 is evaluated in the brackets.) If $S_{1a} > S_{1b} > 0$, then V_0 [10e] lies between V_0 [10d] and V_0 [10c], but if $0 < S_{1a} < S_{1b}$, then V_0 [10c] lies between V_0 [10d] and V_0 [10e]. The relationship between, say, V_0 [10b] and V_0 [10c] depends in general on the magnitude of π^a, S_{1a}, and S_{1b}.

[2] If $\rho^a < \rho^b$, then $\rho^a < \rho_1 < \rho^b$, and it is always true that V_0 [10b] > V_0 [10a], V_0 [10d] > V_0 [10c], and V_0 [10d] > V_0 [10e]. If $S_{1a} > S_{1b} > 0$, then V_0 [10e] lies between V_0 [10c] and V_0 [10d], but if $0 < S_{1a} < S_{1b}$, then V_0 [10c] lies between V_0 [10e] and V_0 [10d].

independent of the number of different projects undertaken.[1] This is a key difference between civilian projects and military projects. In the former, the probability distribution over the possible outcomes on a particular project generally does not depend on whether other distinct projects are undertaken, whereas in the latter, *the existence of many projects dilutes the opponent's ability to commit his limited resources to countering any one of them.* If an opponent has a limited budget for countermeasures and is maximizing his utility subject to that constraint, the introduction of a new project diverts his funds away from countering the original set of projects; the probability of achieving a countermeasure on each of these projects in general will be lower. The rate of return on a particular project is a random variable that is partially under the control of the opponent, but success in countering one project--that is, an outcome in which the rate of return on the investment is low--is likely to be negatively correlated with success in countering the rest of the portfolio. In other words, for a portfolio of military projects, the covariance between the rate of return on a new project and the rate of return on an existing portfolio of projects is likely to be negative.[2] Compare this to a new civilian project. The covariance between the rate of return on a new civilian project and the rate of return on the existing portfolio could be positive, negative or zero, but the larger the original portfolio--that is, the more it resembles national income--the more likely the covariance is to be positive. *Thus, the "pooling" argument often advanced for civilian projects must a fortiori be stronger for military projects.*[3]

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[1] Some might argue that the probability of war is also not independent of the number of projects undertaken.

[2] This is one of the reasons the United States relies on the "triad" for deterrence instead of on a single system. A high probability of developing a countermeasure for one element of the triad is associated with high opponent expenditures on countermeasures. Fewer resources are then available to the opponent to counter the other elements of the triad, resulting in a diminished probability of countering those elements.

[3] This point is worth some amplification. Consider a portfolio of k civilian projects. Let the rate of return on the *ith* project r_i be a random variable. If we invest one dollar in each of the k projects,

To illustrate in a simple model how the ability of the opponent to develop countermeasures that reduce the benefits of a military project can be dealt with analytically, let there be two future states of the world, a and a'. In future state a, no countermeasure is deployed, but in future state a', a countermeasure is deployed. Suppose the benefits in all future periods $j = 1, 2, \ldots, T$ are S if state a occurs and zero if state a' occurs. The current cost of the project is C_0.

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the expected rate of return on the entire portfolio r_p is given by

$$E(r_p) = \frac{1}{k} \sum_{i=1}^{k} E(r_i) \tag{i}$$

and the variance is

$$\mathrm{var}(r_p) = \frac{1}{k^2} \sum_{i=1}^{k} \mathrm{var}(r_i) + \frac{1}{k^2} \sum_{j=1}^{k} \sum_{\substack{i=1 \\ i \neq j}}^{k} \mathrm{cov}(r_i, r_j) \ . \tag{ii}$$

If we add one additional project by investing a (k+1)st dollar, the variance of the rate of return on the new portfolio $r_{p'}$ is

$$\mathrm{var}(r_{p'}) = \left(\frac{k}{k+1}\right)^2 \mathrm{var}(r_p) + \left(\frac{1}{k+1}\right)^2 \mathrm{var}(r_{k+1}) + \frac{2k}{(k+1)^2} \mathrm{cov}(r_p, r_{k+1}) \ . \tag{iii}$$

In other words, the variance of $r_{p'}$ is a weighted average of the variance of r_p, the variance of the new project, and the covariance of the original portfolio with the new project. The variance of $r_{p'}$ will be smaller than that of r_p if and only if

$$\mathrm{var}(r_p) > \alpha\ \mathrm{var}(r_{k+1}) + (1 - \alpha)\ \mathrm{cov}(r_p, r_{k+1}) \tag{iv}$$

where $\alpha = \frac{1}{2k+1}$.

A large and positive covariance term makes Eq. (iv) more difficult to achieve but a negative covariance term almost guarantees the condition will hold for even modest-sized k.

The PCEV is given by:

$$V_0 = -C_0 + S \sum_{j=1}^{T} \left(\frac{\pi_j^a}{\rho_j^a} \right) \tag{11}$$

where π_j^a is the probability that state a will occur in future period j and ρ_j^a is the appropriate discount factor for state a in period j. (π_j^a/ρ_j^a is the price of contingent claims in state a and period j.) The assumption of zero benefits in state a' is very convenient because no assumption about $\rho_j^{a'}$ is needed.

If we make the assumptions that there is a constant probability *in each period* that no countermeasure will be deployed and that the intertemporal marginal rate of substitution in state a is the same between any two sequential periods, then Eq. (11) can be rewritten as:

$$V_0 = -C_0 + S \sum_{j=1}^{T} \left(\frac{\pi}{\rho}\right)^j \tag{11'}$$

where ρ is the intertemporal marginal rate of substitution.

Is it possible to treat the formulation in Eq. (11') *as if* the benefits S are certain, but discounted at ρ plus a risk premium? In other words, is there a δ such that

$$\left(\frac{\pi}{\rho}\right)^j = \left(\frac{1}{\rho + \delta}\right)^j \ ? \tag{12}$$

Equation (12) is easily solved for δ, and one obtains $\delta = \delta(\rho) = \left(\frac{1 - \pi}{\pi}\right)\rho$. The "appropriate risk premium" is proportional to ρ. For example, if $\pi = 0.9$, then $\delta = 0.11\rho$; if $\rho = 0.09$, then $\rho + \delta = 0.10$.

It is easy to demonstrate that if the probability of countermeasure deployment in each period is non-constant over time, then the risk premia in each period are non-constant. If the probability of countermeasure deployment increases in each period, then the appropriate risk premia also increase. Consider the more complex example in which

the probability that no countermeasure will be deployed in period k is given by $e^{-\beta k}$, k = 0, 1, 2, ... This functional form suggests that at the outset the probability of no countermeasure deployment is high but diminishes rapidly as the system matures. For this reason this form is perhaps more useful in the evaluation of military investments. To calculate the risk premium in period j, δ_j, we must solve

$$\frac{\prod_{k=1}^{k=j} e^{-\beta k}}{\rho^j} = \left(\frac{1}{\rho + \delta_j}\right)^j . \tag{13}$$

A closed-form solution is given by:

$$\delta_j = (-1 + e^{\beta(j+1)/2})\rho \tag{14}$$

which means, for example, that in the fifth period, with $\beta = .05$, $\delta_j = .16\rho$; if $\rho = .09$ as in the previous illustration, $\rho + \delta_j = .105$. In general, the appropriate risk premium in period j depends on the probability of countermeasure deployment in each of the j periods.

$$\delta_j = \delta_j(\pi_1, \ldots, \pi_j; \rho) . \tag{15}$$

In summary, the choice of the discount rate is just a part of the larger issue. In evaluating public investments, particularly military projects, states of the world in which tastes, production possibilities, or benefits differ from those of the current state or most likely future state, are too often ignored. It should be possible to improve our assessments of military projects if the effect of possible countermeasures is directly incorporated into the present value calculations.

V. CONCLUSION

The level of our defense effort--that is, what capabilities we wish to have over some appropriate time horizon--should not be governed by the discount rate. The discount rate only helps to choose the most efficient way of obtaining the capabilities we deem necessary. Making a decision on alternative equally effective defense systems based on the time path of costs is precisely like choosing a private investment: The dollars forgone because the wrong alternative was chosen are just as real as the dollars earned by a private investment. Even though Congress may "buy" specific projects, it is important that the menu from which it chooses be weeded of inefficient ways of accomplishing a particular objective. The use of the correct discount rate and discounting procedure is an important part of that process.

One reason why the discussion within the economics profession on the choice of the discount rate has continued without much resolution is that the debate has taken place without much agreement on assumptions. Economists disagree on whether public investment displaces private investment, and if it does, how much is taken away from each sector. There are further disagreements on whether risk is socially avoidable even if it is not privately avoidable, and on how much taxes on capital affect its rate of return. Perhaps a useful product would be a careful array of the working assumptions of the dozen or so major contributions to the discount rate literature.

In Table 1, the recommendations of several economists concerning the nominal rate of discount can be compared. The anticipated rate of inflation in each of these recommendations in unknown but some reasonable numbers can be generated. A *real* rate between 8 and 10 percent can be supported from various studies. The studies by Harberger (1968) and Stockfisch (1969) in which these economists estimated the social opportunity cost of capital are probably the best. In Harberger, the social opportunity cost of capital is a weighted average of the after-personal-income-tax rate of return to savers and the pre-corporate-income-tax cost of capital. Stockfisch calculates the pre-tax rate of

return in several corporate sectors and takes a weighted average of that with the rate of return in the noncorporate sector.

The 10 percent figure suggested by many outstanding economists and enshrined in DoD and OMB directives is an informal estimate of the social opportunity cost of capital. In spite of the back-of-the-envelope nature of the estimate, *careful* studies like the two mentioned above have found 10 percent to be quite reasonable. Furthermore, even though the calculated real opportunity cost of capital may vary slightly from year to year (as a result of federal monetary or fiscal policy), it can only be described from the historical point of view as very stable. The point is that the use of a constant 8 percent or 10 percent as the real discount rate is a lot better than 0 percent.

Defense investments, like private investments, have different payoffs depending on the state of the world that actually materializes. The unique feature of defense investments is that the return is partially under the control of an opponent who is actively seeking to reduce the payoff through countermeasures. (This is in contrast to private investments which face only a benign Nature.) Techniques like the state-preference approach can be used to handle this kind of uncertainty.

In the evaluation of military projects, taking into account alternative states of the world explicitly--especially the benefits (or lack of them) and the valuation of the benefits in these states--should improve defense decisionmaking. In particular, the state-preference approach can be used to analyze the issue of peacetime versus wartime benefits as well as the effect of possible countermeasures.

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